Identity versus Equity Behaviours in a Dynamic Social Network

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Abstract

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Keywords: ??

1 Introduction

Human behaviours in social networks have not been the subject of much experimental research in social psychology. In this paper we study how social interactions amongst multiple actors are shaped by factors such as fairness and equity, reciprocation, ingroup bias and neigbour (nearness) obligations. Each of these factors have been studied extensively in experimental settings that typically eliminate interactions between experimental subjects. This work will attempt to understand how these.....

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Human behaviours in social networks have been the subject of much social-psychology research [1]. In this paper we study social interactions, focussing on the sense of identity of self and others, and the resulting behaviourial dynamics. Complex emotions such as 'playing fair' versus competitivity, reciprocity, or neighbour (nearness) obligations, are possible factors influencing human decision processes in a social environment. This work will attempt to understand how behaviours emerge over time and in response to events in a social network. To study these group interactions, and hence test resulting social-psychological hypotheses, a laboratory-based virtual environment is used. A group of participants interact while playing a game of sequential token (gift) exchanges, over a pre-determined number of rounds. At each round, the participants can observe who donated tokens to whom, and are told the resulting 'wealth' of each player.

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We propose a simple mathematical model to quantify these behaviours between individuals. Data from the 100? virtual games played are presented and are used to test the model??

More complex scenarios are also contemplated, and questions posed. For instance, when participants are divided into two equal-sized teams before the game begins, does their resulting behaviour show a team bias during the token exchanges? Does team bias dominate other behavioural tendencies, such as self-interest or overall fairness to the entire social network?

Finally, suggestions for further research in this field of dynamical social networks are proposed.

2 Motivation

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1. Decision Theory. Interactions, and how experimental traditions ignored interactions, thus compromising internal validity.

2. Nature of experiments.

3. Two competing theories - equity, fairness, justness - versus - self-favouritism and bias.

3 The Game

A group of N participants is gathered in a computer laboratory. Each individual is seated at their own computer, and none have visual access to the other computer screens in the laboratory. The group is told that they will be playing a game of 'token' exchange. The participants are each assigned a different player number, from 1 to N; and everyone is initially given the same number, B, of (virtual) tokens. On the computer screens N numbered dots, corresponding to the N participants, are evenly arranged in a large circle; and beside each dot (virtual player) the value for B appears.

The participants are then told that they will be playing R rounds of the following game. At each round they must give one of their tokens to any of the players on the screen. Only once all the exchanges are privately decided and entered on the individual computers, will a new screen appear. It will show (with the use of arrows) which participants gave a token to which players; and the newly calculated number of tokens owned by the various players will be displayed beside each dot. This is the end of the round. When the group is ready to start the subsequent round, the arrows are deleted, and a new round can begin. Note that at the start of a round, players with zero tokens cannot make a move until they receive token(s) from other players, i.e., a negative number of tokens is not permitted; 'no credit is allowed'. At the end of the final (R^{th}) round, the game is finished.

Technically, the computer display of the rounds of the game consists of a sequence of *graphs*. A graph is a set of vertices (in this situation, the participants) and a set of edges (which, here, correspond to the giving of tokens in a round) joining one vertex to another. If the edges all have a direction, i.e., the flow of giving from one vertex to another is shown by an arrow, then the graph is called a directed graph, or digraph. Thus, in mathematical terms, the picture on the screen in the laboratory would be called a digraph containing N vertices.

4 An Initial Model

In order to provide a mathematical model for the game, we need to define several variables, and form numerous matrices as well as two vectors.
• Let the participants be given labels $p_1, p_2, p_3, \ldots, p_N$.

Since this is a dynamical time system, the superscript, say r, will indicate the round number under consideration, where $1 \le r \le R$.

• Define a vector for the *wealth* (number of tokens) of each of the N participants at the start of round r.

$$\underline{W}^{(r)} := \{w_1,\, w_2,\, w_3,\, \ldots,\, w_N\}$$

gives the number of tokens owned by the various players at the start of round r. So, $\underline{W}^{(1)} = \{\mathrm{B}, \mathrm{B}, \mathrm{B}, \ldots, \mathrm{B}\}$.

Note that since no tokens are lost during the playing of the game, at the end of every round, the sum of the elements, $w_1 + w_2 + \cdots + w_N$, must always equal $B \times N$.

• A vector to count the *change* in the number of tokens for each player, during round r is

$$\underline{c}^{(r)} := \{c_1,\, c_2,\, c_3,\, \ldots,\, c_N\}$$

where each c_j could have a numerical value from -1 (player p_j gave away his token but no tokens were given him during round r) up to $N-1$ (if all the participants, including p_j himself, give their token to p_j.)

At the beginning of each round, the change vector is re-set in order to be ready to accept the moves for the next round. Since players must give one token in each round, usually, $\underline{c}^{(r)} = \{-1,\, -1,\, -1,\, \ldots,\, -1\}$ at the start of round r. The only exception would be if a player, say p_u finished the previous round with zero tokens, and so is 'broke' and thus unable to give away a token. Then, c_u would be set to zero, rather than (-1). For example, if $N = 4$, $B = 10$ and $\underline{W}^{(12)} = \{5,\, 29,\, 0,\, 6\}$, the initial change vector for round 12 would be $\underline{c}^{(12)} = \{-1,\, -1,\, 0,\, -1\}$.

• Finally, an $N \times N$ matrix, $E^{(r)}$, will model the *empathy* felt by each player towards the others in the group, at the start of round r. The matrix entry $e_{i\,j}$ (in row i and column j of the matrix) will represent how much (**column**) player p_j 'likes' (**row**) player p_i, and hence this matrix gives a numerical quantification of the attitude, or affinity each player feels towards the others in the game. These numbers determine the moves of each player in that round.

4.1 Formulating the Empathy Matrix

Some emotive factors which might influence the token-giving choice of each player could be

1. Their sense of fairness (F), viz., concern for the underdog (a player with very few tokens at that time).

2. The desire to reciprocate (Rec) a token given in the previous round.

3. A propensity to award a neighbouring player, that is, one in their vicinity (V) (a closer dot on the computer screen).

These factors will be combined to create the entries in the empathy matrix.

To determine the entries for $F^{(r)}$, the fairness matrix in round r, consider the wealth of each individual player at the end of round $r-1$. Rank the players from poorest to richest, then assign decreasing fairness weights. So, the poorest player receives weight N, and the richest, weight 1. These various weights are then entered in the matrix F. Assuming that all players in the game will give the same fairness ranking to player p_i, we have that every cell in row i of the matrix would contain the same value, the fairness weight for player p_i.
For instance, for the example above, where $\underline{W}^{(12)} = \{5, 29, 0, 6\}$, the fairness matrix would be $F^{(12)} = \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 \\ 2 & 2 & 2 & 2 \end{pmatrix}$. If two or more players have the same wealth, they would be assigned the same fairness weighting, and the weightings would decrease accordingly. Thus, if $\underline{W}^{(5)} = \{7, 20, 7, 6\}$, the corresponding fairness matrix would contain the numbers 3, 1, 3, and 4 in its first, second, third and fourth rows, respectively.

The second factor in determining player affinity is a feeling of gratitude for being given a token from another player in the previous round. The reciprocity matrix, $Rec^{(r)}$, would reflect back the exchanges made in round $r-1$. If in the previous round player p_i gave his token to player p_j, then the reciprocity desire would suggest that p_j should be happy to give his token to p_i in the present round. Visually, if the digraph in the previous round had an arrow from p_i's dot into p_j's dot, then, in the next round, p_j would like to reciprocate and send his arrow (give his token) into p_i's dot. Some players might have received more than one arrow (token) in the previous round, and so would desire to reciprocate to all those donors. If, however, a player self-gave in the previous round, we would not expect this desire to be repeated in the next round. Therefore, formulating the reciprocity matrix, the entry $rec_{i,j}$, for $i \neq j$, would be 1 if p_i gave a token to p_j in the previous round; and otherwise, it would be 0.
As an example, if in round 6, p_1 gave his token to p_4, p_2 gave his token to p_3, p_3 gave his token to p_4, and p_4 gave his token to himself, then $Rec^{(7)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

The vicinity matrix, V, will contain the numerical values determined by the nearness factor. The *distance* between two players is the length of the shorter path from one player (dot) to the other, going around the circle. So, neighbours would have distance 1 between them; neighbours of neighbours distance 2, etc. Closer players would be assigned a larger vicinity factor, while more distant players get a progressively lower vicinity value. Players are not considered neighbours of themselves, so the main diagonal of the matrix would contain all zeros. Entries $v_{i,j}$ in the vicinity matrix would be determined by

$$v_{i,j} = \begin{cases} N - 1 - \text{distance between } p_i \text{ and } p_j & \text{when } i \neq j \\ 0 & \text{when } i = j. \end{cases}$$

For example, when $N = 6$ players, the vicinity matrix is $V = \begin{pmatrix} 0 & 4 & 3 & 2 & 3 & 4 \\ 4 & 0 & 4 & 3 & 2 & 3 \\ 3 & 4 & 0 & 4 & 3 & 2 \\ 2 & 3 & 4 & 0 & 4 & 3 \\ 3 & 2 & 3 & 4 & 0 & 4 \\ 4 & 3 & 2 & 3 & 4 & 0 \end{pmatrix}$.

Since positions of the players (the dots on the screen) do not move during game play, this matrix, V, will remain the same for the entire game.

Entries in the empathy matrix would then be the sum of these three matrix values.

$$E^{(r)} = F^{(r)} + Rec^{(r)} + V. \tag{1}$$

Note that, in general, $e_{i\,j} \neq e_{j\,i}$ since the amount of empathy player p_j feels for player p_i might not be equally reciprocated. In mathematical terms, we say that the matrix E is not symmetric.

The values $e_{j\,j}$, which lie on the main diagonal of the matrix, show the self-liking of player p_j and would give an indication of their level of self-interest, and hence their tendency towards self-giving, at that point in the game. When a player decides to self-give in a round, it is illustrated on the computer screen as a loop, rather than an arrow.

In the first round of the game, fairness and reciprocity issues do not arise, so the initial empathy matrix, $E^{(1)}$, would be determined only by the vicinity condition. Thereafter, the N^2 values in the empathy matrix will change for each new round, reflecting the players' changing levels of affinity towards the different participants, as the game progresses.

Thus, for $r \geq 2$, all three factors, $F^{(r)}$, $Rec^{(r)}$ and V, will play a role in forming $E^{(r)}$.

4.2 The Rules of the Game

The following are the transitional rules for stepping from one discrete time period (round) to the next.

Rule 1: *In the first round, players feel more empathy for their neighbours in the game. They choose a neighbour at random, and give them the first token.*

The initial empathy matrix, $E^{(1)}$, will be equal to the vicinity matrix V. The preferred player would be randomly chosen from one of his two neighbours. Thus, the entries in $\underline{c}^{(1)}$ would be altered from the initial $\underline{c}^{(1)} = \{-1, -1, -1, \ldots, -1\}$ setting. Some entries might remain at (-1); however, the sum of the resultant entries in this vector must be 0, since no tokens are ever lost during play. The result would be the change vector to be used in the first round.

Rule 2: *The most-liked player will be given the token.*

Perform the following iterative procedure, for column j ranging from 1 to N.

For each player p_j, find his preferred player, p_k (k could be equal to j) from the empathy matrix: then p_j will give his token to p_k in the present round. That is, if

$$\text{Maximum } \{e_{1\,j},\, e_{2\,j},\, e_{3\,j},\, \ldots,\, e_{N\,j}\} = e_{k\,j} \tag{2}$$

then replace the current c_k value by $c_k + 1$.

When the last player (column) has been considered ($j = N$), then the new wealth vector can be formed as follows:

$$\underline{W}^{(r)} = \underline{W}^{(r-1)} + \underline{c}^{(r-1)}, \tag{3}$$

for $r \geq 2$.

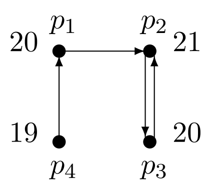
Player p_j already had w_j tokens from the previous round, and in this round he gains c_j tokens.

Rule 3: *If two or more players are equally 'liked', break the tie by giving the token to the 'liked' player with the fewest number of tokens at that moment. Furthermore, if there is more than one such poorest 'liked' player, break this second tie with a random choice.*

4.3 Some Small Examples

For an illustration of how the transitional rules work, consider the following small example, with $N = 4$ and $B = 20$. Initially, $\underline{c}^{(1)} = \{-1, -1, -1, -1\}$. Random choice of neighbours (Rule 1) in the first token exchange might result in p_1 giving his token to p_2, p_2 to p_3, p_3 to p_2, and p_4 to p_1, as indicated by the bold-face digits in the empathy matrix below. Players p_2 and p_3 exchanged tokens, though of course neither knew the move the other would be making during the round. At the end of the first round, the players' wealth would be $w_1 = 20$, $w_2 = 21$, $w_3 = 20$, and $w_4 = 19$. See the illustration below for the steps taken in this first round.

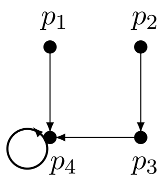
$$\underline{W}^{(1)} = \begin{matrix} 20 \\ 20 \\ 20 \\ 20 \end{matrix} \qquad E^{(1)} = V = \begin{pmatrix} 0 & 2 & 1 & \mathbf{2} \\ \mathbf{2} & 0 & \mathbf{2} & 1 \\ 1 & \mathbf{2} & 0 & 2 \\ 2 & 1 & 2 & 0 \end{pmatrix} \qquad \underline{c}^{(1)} = \begin{matrix} 0 \\ 1 \\ 0 \\ -1 \end{matrix}$$



$$\underline{W}^{(2)} = \begin{matrix} 20 \\ 21 \\ 20 \\ 19 \end{matrix}$$

Observe that the sum of the entries in the change vector is zero, as expected. Note also that the sum of the entries in $\underline{W}^{(2)}$ is $20+21+20+19 = 80 = 20 \times 4 = B \times N$, since no tokens were lost during play. At the end of round 1, the arrowed edges are removed from the digraph, and round 2 can begin, with $\underline{W}^{(2)} = \{20,\ 21,\ 20,\ 19\}$.

As a second example, consider playing the virtual game again, with the same values for N and B, and suppose that in round 6, p_1 gave his token to p_4, p_2 gave his token to p_3, p_3 gave his token to p_4, and p_4 gave his token to himself. So, the digraph in round 6 was



Observe that in this sixth round, participant p_4 gave himself the token, which is illustrated by the 'loop' in the digraph. Furthermore, suppose that at the end of round 6 (and hence the start of round 7), $\underline{W}^{(7)} = \{18,\ 23,\ 22,\ 17\}$. Equipped with this information, round 7 can begin. We find

$$F^{(7)} = \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \end{pmatrix}, \qquad Rec^{(7)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

and the vicinity matrix is unchanged, so will be the same as in the first example (but without the bold-facing). Form the empathy matrix using formula (1).

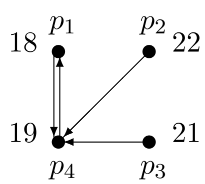
$$\begin{aligned} E^{(7)} &= F^{(7)} + Rec^{(7)} + V \\ &= \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 2 & 1 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 0 & 2 \\ 2 & 1 & 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 5 & 4 & 6 \\ 3 & 1 & 4 & 2 \\ 3 & 4 & 2 & 5 \\ 6 & 5 & 6 & 4 \end{pmatrix}. \end{aligned}$$

The round would play out as indicated below, using equations (2) and (3) of Rule 2.

$\underline{W}^{(7)}$: 18, 23, 22, 17

$$E^{(7)} = \begin{pmatrix} 3 & \mathit{5} & 4 & \mathbf{6} \\ 3 & 1 & 4 & 2 \\ 3 & 4 & 2 & 5 \\ \mathbf{6} & \underline{\mathit{5}} & \mathbf{6} & 4 \end{pmatrix}$$

$\underline{c}^{(7)}$: 0, -1, -1, 2



$\underline{W}^{(8)}$: 18, 22, 21, 19

Observe that player p_2 felt equal affinity (value of 5) for players p_1 and p_4. To break this tie, Rule 3 considers the present wealth of these two players. Since p_1 had 18 tokens at the start of the round, and p_4 had only 17 tokens, player p_2 will give his token to player p_4, as indicated by the underlined value in the empathy matrix.

Continuing this same game, the wealth vectors for the next two rounds are $\underline{W}^{(9)} = \{20, 21, 20, 19\}$, and then $\underline{W}^{(10)} = \{20, 20, 19, 21\}$.

5 A More Complex Model

Having understood the mechanisms of the initial model, and seen how the transitional rules determine the steps from one round to the next, we now consider some fine-tuning. We require a more complex formulation of the empathy matrix, to reflect other human behaviours. For instance

- The starting empathy matrix, $E^{(1)}$ $(= V)$, was defined to contain zeros on the main diagonal. However, it might be reasonable to imagine that the main diagonal entries could contain lower (or higher) numbers, to reflect an initial selfless (or superiority) feeling of each player towards himself.
- The reciprocity matrix definition has been assuming a 'one-step memory', i.e., that players only recall moves from the most recently played round, rather than earlier rounds as well. A modification of the definition of $Rec^{(r)}$ could allow for a 'two (or more)-step memory'.

Of more importance, however, is the fact that thus far in the model there has been too little attention given to the second theory of human dynamics in sociology, that of self-favouritism, or greed.????? Indeed, when the empathy matrix only considers the three factors of fairness, reciprocity and nearness, initial examples seem to indicate that the game quickly converges to an equilibrium state, with the wealth of all participants moving close to B, and varying little for the rest of the game. This unlikely scenario could be due to the small N values used in the examples - with more participants, other dynamics could manifest themselves. Another possible explanation is the strong presence of equity factors, without balancing the terms in the empathy matrix with some favouritism considerations, such as self-interest or personal bias.

Hence, consider the following modified game.

Before the N participants begin their computer laboratory game, they are (randomly) divided into two equal subgroups, or teams. (This would require N to be an even number.) The players would not know which people in the laboratory are in their group, but the dots on the screen would be coloured alternately in two different colours, and the players would thus know, from the start, to which group they belonged, and could differentiate the dots as belonging to their group, or the other group. With this additional dynamic, let us consider a more complex empathy matrix which reflects both the equity and the selfishness behaviours in humans.

5.1 Re-formulating the Empathy Matrix

Add to our list of emotive factors influencing the token-giving choice of each player, the following

4. An in-group bias (G), that is, a preference to give a token to a member in the same group.

5. A tendency for self-interest (S), or greed, to award a token to oneself.

The in-group membership matrix G will contain numerical values determined by the membership of the participant in the same or other group. Its entries $g_{i,j}$ are given by

$$g_{i,j} = \begin{cases} 1 & \text{when } p_i \text{ and } p_j \text{ are in the same group,} \\ 0 & \text{when } p_i \text{ and } p_j \text{ are in different groups.} \end{cases}$$

Since the dots on the computer screen are alternately coloured, we would have, for example a group matrix $G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$ when there are $N = 6$ players. Since group membership is fixed for the duration of the game, this matrix, like V, will remain the same for the entire game.

Self-interest is easily modelled by a diagonal matrix, S, with 1's on the main diagonal entries ($s_{j,j}$) and zeros in all other entries. Again, this matrix would remain the same for each round.

With these added factors, we could replace Rule 1 with

Rule $1'$: *In the first round, players feel more empathy towards members in their own group. So, players randomly give their first token to a member of their assigned group, including possibly self-giving.*

Thus, $E^{(1)}$ would now be equal to G.

The neighbourhood affinity matrix V could now be replaced with the group and self-interest matrices. Thus, for $r > 1$, we would replace formula (1), with

$$E^{(r)} = F^{(r)} + Rec^{(r)} + G + S. \tag{4}$$

Returning to our second hypothetical example, still using $N = 4$, $B = 20$ and supposing that at the end of round 6 (and hence the start of round 7), the wealth vector was $\underline{W}^{(7)} = \{18,\ 23,\ 22,\ 17\}$, our new empathy matrix for round 7 would be calculated as

$$\begin{aligned} E^{(7)} &= F^{(7)} + Rec^{(7)} + G + S \\ &= \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{5} & 3 & \mathit{4} & 4 \\ 1 & 3 & 2 & 2 \\ 3 & 2 & 4 & 3 \\ 4 & \mathbf{5} & \underline{\mathit{4}} & \mathbf{6} \end{pmatrix}. \end{aligned}$$

So, in round 7, players p_1 and p_4 will give their tokens to themselves, p_2 gives his token to p_4, and p_3 uses Rule 3 to break the tie between players 1 and 4 and opts to give his token to p_4. The change vector will be $\underline{c}^{(7)} = \{0, -1, -1, 2\}$ and the result at the end of the round will give the wealth vector $\underline{W}^{(8)} = \{18, 22, 21, 19\}$.

This result is different from the initial model. Indeed, there is now some self-giving (the influence of the S matrix), with the in-group bias (matrix G) stopping the exchange of tokens between players 1 and 4, who are neighbours but are assigned to different groups. However, the weight of the group bias is still not very strong, since player 3, when choosing to break the tie, decides to give his token to a member in the other group.

In order to study the fine balance between the two contrasting sociological ????? theories, a *weight coefficient* could be applied to each of the 4 matrices, to vary the importance given to the various factors. (Ultimately, our aim would be to determine what numerical values of these weights would best mimic human behaviour.) Thus, we refine equation (4) to

$$E^{(r)} \;=\; \alpha_f\, F^{(r)} \,+\, \alpha_r\, Rec^{(r)} \,+\, \alpha_g\, G \,+\, \alpha_s\, S \qquad (5)$$

where the α coefficients are non-negative integers.[1]

Let us again play round 7 of the example above, with, say $\alpha_f = 2$, $\alpha_r = 1$, $\alpha_g = 4$, and $\alpha_s = 1$. The empathy matrix would be calculated as

$$\begin{aligned}
E^{(7)} &= 2\times F^{(7)} \,+\, 1\times Rec^{(7)} \,+\, 4\times G \,+\, 1\times S \\
&= 2\times\begin{pmatrix} 3&3&3&3\\ 1&1&1&1\\ 2&2&2&2\\ 4&4&4&4 \end{pmatrix} + \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix} + 4\times\begin{pmatrix} 1&0&1&0\\ 0&1&0&1\\ 1&0&1&0\\ 0&1&0&1 \end{pmatrix} + \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} \\
&= \begin{pmatrix} 6&6&6&6\\ 2&2&2&2\\ 4&4&4&4\\ 8&8&8&8 \end{pmatrix} + \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix} + \begin{pmatrix} 4&0&4&0\\ 0&4&0&4\\ 4&0&4&0\\ 0&4&0&4 \end{pmatrix} + \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{11}&6&\mathbf{10}&7\\ 2&7&3&6\\ 8&4&9&5\\ 8&\mathbf{12}&8&\mathbf{13} \end{pmatrix}.
\end{aligned}$$

Here a weight of 4 is given to the group matrix, and a weight of two for the fairness term, and the other factors have weight one. Once again the first and last player will self-give, player 2 still gives to player 4 (same group), but now player 3 has chosen to give his token

[1] question from Jane: What would be the best notation here? Using a, b, c, and d is not very illustrative/intuitive, nor is α, β, γ, and δ. At least using the subscripts f, r, g, s on, say, α, the corresponding matrix is obvious.

to the same group member, p_1. So, the heavier weight on G has given the in-group bias more influence in the empathy matrix.

The change vector will be $\underline{c}^{(7)} = \{1, -1, -1, 1\}$ and the result at the end of the round will give the wealth vector $\underline{W}^{(8)} = \{19, 22, 21, 18\}$.

Further calculations show that the same result will occur for any value $\alpha_g \geq 4$, if the other three coefficients are unchanged.

As a final illustration of this refined empathy formula, let us take, say, $\alpha_f = 2$, $\alpha_r = 1$, $\alpha_g = 4$, and $\alpha_s = 3$. The empathy matrix in round 7 would be calculated as

$$
\begin{aligned}
E^{(7)} &= 2 \times F^{(7)} + 1 \times Rec^{(7)} + 4 \times G + 3 \times S \\
&= 2 \times \begin{pmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + 4 \times \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} + 3 \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 6 & 6 & 6 & 6 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 8 & 8 & 8 & 8 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 4 & 0 \\ 0 & 4 & 0 & 4 \\ 4 & 0 & 4 & 0 \\ 0 & 4 & 0 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{13} & 6 & 10 & 7 \\ 2 & 9 & 3 & 6 \\ 8 & 4 & \mathbf{11} & 5 \\ 8 & \mathbf{12} & 8 & \mathbf{15} \end{pmatrix}.
\end{aligned}
$$

More weight (a coefficient of 3, instead of 1) is given to the self-interest factor, and now three players self-give (players 1, 3, and 4), and player 2 still gives to player 4 (same group). The change vector will be $\underline{c}^{(7)} = \{0, -1, 0, 1\}$ and the result at the end of the round will give the wealth vector $\underline{W}^{(8)} = \{18, 22, 22, 18\}$.

6 Simulation Results

In order to find the values of the weight coefficients which most closely describe human dynamics in a social setting, we ran computer simulations, for $N = 14$ players, and $R = ?$ rounds, since these were the values used in the virtual environment games run in the laboratory. Various values for the α coefficients were tried, with the aim of matching the simulation outputs to the results from the human experiments. ???? ?????? TO BE CONTINUED....

7 Experimental Results

PMB

8 Analysis of Model/ Discussion

PMB INPUT HERE

What value should be given to R, the number of rounds? If it is too big, the players could get tired, lose interest and might play at random, rather than thinking through their feelings. On the other hand, if there are too few rounds, then the group dynamics may not have developed sufficiently. The lab tests had $R =?$, because...

How many participants should be chosen to play the game, viz., what is the optimal value for N? The computer laboratory logistics could limit the number of players who can be accommodated at one time. However, there is a risk that if a large number of computers are made available, no player empathies will form, as there will be too many dots on the screen. On the other hand, too few participants could result in uninteresting data, and with little variation in the final outcomes. The data collected in the experiments all have $N =?$, since...

discuss the psychology of the decisions here?

9 Conclusions

TBA!

References

[1] Klein, Spears, Reicher 2007.